MATH 1105 - FALL 2008 - 9-22-08
SECTION 2
QUIZ 1

You have 25 minutes to complete the following problems. No notes or calculators are allowed or necessary. This quiz is out of 30 points. Good Luck!

Problem 1 (7 Points): Choose and complete **ONE** of the following two problems:

A. Find the inverse of

$$\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}.$$

$$\left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 2 & 0 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{2} & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

Hence,

$$\begin{bmatrix} \frac{3}{2} & 1 \\ 1 & 1 \end{bmatrix} \text{ is the inverse of } \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}.$$

B. 6 year old Julie runs a lemonade stand. The cost function of the lemonade stand is $C(x) = 20 + .1x$, and the revenue function is $R(x) = .5x$ where x is the number of 12 oz lemonades that Julie makes and $C(x)$ and $R(x)$ are in dollars. Assuming that Julie sells every glass of lemonade she makes, how many lemonades does Julie need to make in order to break-even? You may leave your answer in the form of a fraction.

$C(x) = R(x)$ implies $20 + .1x = .5x$, yielding $\frac{20}{.4} = 50 = x$. Julie needs to make fifty 12 oz glasses of lemonade in order to break-even.

Problem 2 (9 Points): Are the following 3 statements true or false? Only answer true if the statement is always true under the stated conditions, otherwise answer false. You do not need to explain your answer.

A: Suppose L is a line in the x-y plane, and that L is not vertical. True or False: The slope L is defined.

True, slope is defined for all non-vertical lines by choosing two points (x_1, y_1) and (x_2, y_2) on the line and calculating $\frac{y_2 - y_1}{x_2 - x_1}$.

B: Suppose M is a matrix with the same number of rows and columns. True or False: M has an inverse.

False, for example

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

has no inverse.

C: Suppose E and F are events in a given sample space of equally likely outcomes S. True or False: $P(E \cup F) = P(E) + P(F)$.

False, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Problem 3 (7 Points): If applying the Gauss-Jordan method to the matrix

$$\left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right]$$

yields the matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right],$$

then at what point(s) do the lines $ax + by = c$ and $dx + ey = f$ intersect?

The intersection point is $(2, -1)$.

Problem 4 (7 Points): Consider a sample space of equally likely outcomes S, and events E and F, $E \subseteq S$, $F \subseteq S$. The Venn Diagram for these sets is shown below. Calculate $P(E)$ and $P(F)$.

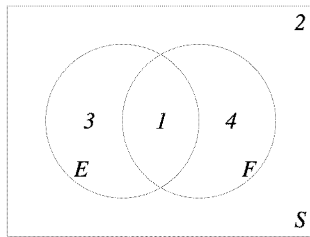

FIGURE 1. Venn Diagram for Problem 4

$n(S) = 1 + 2 + 3 + 4 = 10$, $n(E) = 1 + 3 = 4$, $n(F) = 1 + 4 = 5$.
$P(E) = \frac{n(E)}{n(S)} = \frac{4}{10} = .4$, and $P(F) = \frac{n(F)}{n(S)} = \frac{5}{10} = .5$

Bonus (3 Points): A survey was taken of 20 people to see if they like cats and dogs. Here are some of the results of the survey:

10 people like cats
12 people like dogs
4 people like neither cats nor dogs

How many of the 20 people surveyed like both cats and dogs?

$x =$ number of people that like cats and not dogs
$y =$ number of people that like cats and dogs
$z =$ number of people that like dogs and not cats

The data above yields the following system of equations:
$x + y = 10$
$y + z = 12$
$x + y + z + 4 = 20$

By substitution, $x + 16 = 20$ implies $x = 4$. Further substitution gives $y = 6$ and $z = 6$. Hence, 6 of the 20 people like both cats and dogs.